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13. ABSTRACT (Maximum 200 words)

A new solution technique for general periodic systems encountered in many ... systems including the helicopter rotor-blade dynamics has been developed. In this technique, the state transition matrices (STM) of periodic systems are obtained in terms of the shifted Chebyshev polynomials of first and second kind. Due to the excellent convergence properties of Chebyshev polynomials, the approach is found to be super efficient in terms of the CPU time with accuracy level comparable to any higher order numerical algorithms such as Runge-Kutta, Adams-Mouton, etc., schemes. The technique is suitable for both numerical and symbolic implementations. The method can be used in a variety of applications such as stability of linear and nonlinear periodic systems, response calculations of linear and nonlinear periodic systems, design of control systems for periodic systems, direct determination of periodic orbits of nonlinear systems and nonlinear analysis of periodic systems in stable, center and unstable manifolds. Case studies corresponding to each of these applications have been reported. Apart from the above mentioned utilities, the technique has resulted in a very practical procedure in obtaining the well-known Liapunov-Floquet (L-F) Transformation matrix which allows one to design a control system in the time-invariant domain. Currently, the research efforts are directed toward utilizing the L-F transformation matrix for various other problems such as order reduction, bifurcation analysis and nonlinear control design strategies for periodic systems. In the following, a short enumeration of the achievements of the project is presented.

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Summary

A new solution technique for general periodic systems encountered in many mechanical systems including the helicopter rotor-blade dynamics has been developed. In this technique, the state transition matrices (STM) of periodic systems are obtained in terms of the shifted Chebyshev polynomials of first and second kind. Due to the excellent convergence properties of Chebyshev polynomials, the approach is found to be super efficient in terms of the CPU time with accuracy level comparable to any higher order numerical algorithms such as Runge-Kutta, Adams-Mouton, etc., schemes. The technique is suitable for both numerical and symbolic implementations. The method can be used in a variety of applications such as stability of linear and nonlinear periodic systems, response calculations of linear and nonlinear periodic systems, design of control systems for periodic systems, direct determination of periodic orbits of nonlinear systems and nonlinear analysis of periodic systems in stable, center and unstable manifolds. Case studies corresponding to each of these applications have been reported. Apart from the above mentioned utilities, the technique has resulted in a very practical procedure in obtaining the well-known Liapunov-Floquet (L-F) Transformation matrix which allows one to design a control system in the time-invariant domain. Currently, the research efforts are directed toward utilizing the L-F transformation matrix for various other problems such as order reduction, bifurcation analysis and nonlinear control design strategies for periodic systems. In the following, a short enumeration of the achievements of the project is presented.

Development of a New Computational Scheme:

The computational scheme is based on the idea that the state vector and the periodic matrix of linear/periodic system can be expanded in terms of Chebyshev polynomials over the

principal period. Such an expansion reduces the original problem to a set of linear algebraic equations from which the solution in the interval of one period can be obtained. Combined with the Floquet theory, the STM at the end of the one period provides the stability conditions. Two formulations are developed. The first formulation is suitable for equations in the state space form while the second can be applied directly to a set of second order equations and is called the *direct formulation* method. The direct formulation method is found to be the fastest of all numerical techniques available today.

Linear and Nonlinear Analysis:

The scheme developed above is applied to a variety of practical illustrative examples. The linear analyses include systems represented by the Mathieu equation [1]*, a parametrically excited fixed-fixed column[1], single and multibladed (up to five blades) rotors in forward flight [6] and rotor-bearing systems [13]. In all cases the technique is found to be efficient and accurate when compared to the Runge-Kutta or similar numerical algorithms. Also, the technique is found suitable for constructing either numerical or approximate analytical solutions. Stability charts of Mathieu equation obtained via symbolic manipulation using this technique [3, 14] is accurate and comparable to similar charts obtained by numerical methods. The results from the various studies indicate that the suggested approach is by far the most efficient one, particularly for systems with larger dimensions. The technique combined with Picard-type iterative procedure yields the solutions of nonlinear periodic systems as well [5, 10]. Such an approach is found to duplicate the periodic, multi-periodic and chaotic solutions of nonlinear periodic systems. The technique does not require the inversion of the mass matrix in the case

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*Indicates paper number under 'List of Publications'

of multi-degrees-of-freedom systems. Further, the approach provides a procedure for determining the periodic solutions of the nonlinear periodic systems directly and has resulted in an algorithm known as the *Direct Determination of Periodic Solutions* (DDPS). In all the nonlinear cases studied, the technique is found to retain the efficiency character throughout without any problems.

Control of Linear Periodic Systems:

A new methodology has also evolved in the study of control of linear periodic systems following the developments of the above mentioned scheme [4, 11]. Controllers using the full-state feedback and output state feedback are designed. The designs are accomplished using an algebraic method incorporating Chebyshev polynomials coupled with the principles of optimal control theory. The computational characteristics of this scheme has been studied by considering one through five mass inverted pendulums subjected to periodic loads. As the size of the system becomes larger and larger, the efficiency of the scheme is found to improve. The new design procedure is found to be efficient, simple and straight forward in the design of controllers for periodically time-varying linear systems. Unlike the existing controller design procedures which require two levels of iterations, the scheme presented here has just one level of controller design. Further, the controller designs using the time-invariant methods have also been carried out through the application of the L-F transformation matrix[7, 16].

Analysis of Quasilinear Periodic Dynamical Systems:

As a result of these developments, a new analysis technique in the study of general quasilinear systems with periodically varying coefficients has also evolved through the

application of the Liapunov-Floquet Transformation matrix [8, 12]. The method is based on the fact that all quasilinear periodic systems can be replaced by similar systems whose linear parts are time-invariant via the L-F transformation. Once the transformation has been applied, the solution of the resulting system is obtained through an application of the *time-dependent normal form* theory. The method is suitable for both numerical and symbolic computations and in some cases closed form solutions have also been obtained. It is shown that this technique is virtually devoid of the small parameter restriction unlike the averaging and perturbation procedures. The method is found to be applicable even to those systems for which the generating solutions do not exist in the classical sense.

Epilogue:

The scheme developed for the stability analysis of periodic systems has given rise to many practical applications in the study of dynamical systems with periodic coefficients. A set of new strategies are being developed to deal with many aspects of the analysis, encompassing both linear and nonlinear periodic systems. The project has provided a totally new perspective in tackling stability, analysis, control and bifurcations of periodically varying dynamical systems. Not withstanding such a variety of applications, the solution method is also applicable to boundary value problems associated with the periodic differential equations[2].

LIST OF PUBLICATIONS

Archival Publications:

a) Publications already in print:

1. S.C. Sinha and D. -H. Wu, "An Efficient Computational Scheme for the Analysis of Periodic Systems", *J. Sound and Vibration*, 151(1991), pp. 91-117.

2. S.C. Sinha, T. -S. Liu and N.R. Senthilnathan, "A New Computational Technique for the Stability Analysis of Slender Rods", *Archives of Applied Mechanics*, 62(1992) pp. 347-360.

3. S.C. Sinha, D. -H. Wu, V. Juneja and P. Joseph, "Analysis of Dynamic Systems with Periodically Varying Parameters via Chebyshev Polynomials, *Journal of Vibration and Acoustics*, 115 (1993), pp. 96-102.

b) Accepted for Publication:

4. Paul Joseph, R. Pandiyan and S.C. Sinha, "Optimal Control of Mechanical Systems Subjected to Periodic Loading via Chebyshev Polynomials, <u>To appear</u> *Optimal Control Applications & Methods*, 1993.

5. S.C. Sinha, N.R. Senthilnathan and R. Pandiyan, "A New Numerical Technique for the Analysis of Parametrically Excited Nonlinear System", <u>To appear</u> in *Nonlinear Dynamics*, an international journal, 1993.

6. D. -H. Wu and S.C. Sinha, "A New Approach in the Analysis of Linear Systems with Periodic Coefficients with Application in Rotorcraft Dynamics", <u>To appear</u> in *The Aeronautical Journal*, 1993 or 1994.

c) Submitted for Publication

7. S.C. Sinha and P. Joseph, "Control of General Dynamical Systems with Periodically Varying Parameters via Liapunov-Floquet Transformation", <u>Submitted to</u> the *Journal of Dynamic Systems, Measurement, and Control*

8. S.C. Sinha and R. Pandiyan, "Analysis of Quasilinear Dynamical Systems with Periodic Coefficients via Liapunov-Floquet Transformation", <u>Submitted to</u> *International Journal of Non-Linear Mechanics*

3b. Proceedings and Presentations at Professional Conferences:

9. S.C. Sinha, D. -H. Wu, V. Juneja and P. Joseph, "Analysis of Dynamic Systems with Periodically Varying Parameters via Chebyshev Polynomials", Presented at the 13th Biennial ASME Conference on Mechanical Vibration and Noise, September 22-25, 1991, Miami, DE-Vol 34, pp. 225-233, 1991.

10. N.R. Senthilnathan and S.C. Sinha, "Analysis of Nonlinear Periodic Systems via Chebyshev Polynomials", 4th Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, June 7-11, 1992, VPISU, Blackburg, VA.

11. P. Joseph, R. Pandiyan and S.C. Sinha, "On the Optimal Control of Large-Scale Mechanical Systems Subjected to Periodic Loading", 4th Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, June 7-11, 1992, VPISU, Blacksburg, VA.

12. J.S. Bibb and S.C. Sinha, "Computation and Applications of Liapunov-Floquet Transformation Matrices for General Periodic Systems", 4th Conference on Nonlinear Vibrations, Stability, Dynamics of Structures and Mechanisms, June 7-11, 1992, VPISU, Blacksburg, VA.

13. S.C. Sinha, D-. H. Wu and T.K. Chen, "An Efficient Computational Scheme for the Dynamic Analysis of Anisotropic Rotor-Bearing Systems", International Congress on Recent Developments in Air-And Structure-Borne Sound and Vibration, March 6-8, 1990, Auburn University, Auburn, AL 36849. Proceedings pp. 909-917.

14. S.C. Sinha and V. Juneja, "An Approximate Analytical Solution for Systems with Periodic Coefficients via Symbolic Computation", AIAA/ASME/ASCE/AHA/ASC 32nd Structures, Structural Dynamics and Material Conference, April 8-10, 1991, A Collection of Papers, Part 1, pp. 790-797.

15. R. Pandiyan and S.C. Sinha, "On the Analysis of Dynamical Systems with Periodically Varying Parameters via Liapunov-Floquet Transformation", to be presented in the Symposium on *Dynamics and Vibration of Time-Varying Systems and Structures*, 14th ASME Biennial Conference on Mechanical Vibration and Noise, September 19-22, 1993, Albuquerque, NM.

16. P. Joseph and S.C. Sinha, "Control of Periodically Varying Dynamical Systems via Time-Invariant Technique", To be presented in the Symposium on *Dynamics and Vibration of Time-Varying Systems and Structures*, 14th ASME Biennial Conference on Mechanical Vibration and Noise, September 19-22, 1993, Albuquerque, NM.

M.S. and Ph.D. Degrees Awarded

Name	Degree Awarded	Year	Citizenship
Der-Ho Wu	PhD	1991	Non US
Paul Joseph	PhD	August 1993	Permanent Resident
R. Pandiyan	PhD	January 1994	Non US
T. K. Chen	MS	1990	Non US
Vikas Juneja	MS	1991	Non US
John S. Bibb	MS	1992	US Citizen